FIN5CRM – Modeling Additional Problems on CDSs – Solution

1- The unconditional default and survival probabilities, are

Time (years)	*Default Probability*	*Survival Probability*
1	0.0300	0.9700
2	0.0291	0.9409
3	0.0282	0.9127
4	0.0274	0.8853
5	0.0266	0.8587

Calculation notes:

Year 1: Survival probability = 1-3%.
Year 2: Marginal Default probability = 3% x 97%
Survival probability = $97\%^2$
Year 5: Marginal Default probability = 3% x 88.53%
Survival probability = $88.53\%^5$

The present value of the expected regular payments by the buyer of protection (payment rate is s per year) and the notional principal is $1, are

Time (yrs)	*Probability of survival*	*Expected Payment*	*Discount Factor*	*PV of Expected Payment*
1	0.9700	0.9700s	0.9324	0.9044s
2	0.9409	0.9409s	0.8694	0.8180s
3	0.9127	0.9127s	0.8106	0.7398s
4	0.8853	0.8853s	0.7558	0.6691s
5	0.8587	0.8587s	0.7047	0.6051s
Total				3.7364s

Discount factor year yr t = $e^{-7\% \times t}$ and so on …
PV of Expected Payment = *Expected Payment* x *Discount Factor.*

Note that it is assumed that defaults always occur mid-year.

The present value of the expected payoffs received by protection buyer (notional principal =$1), are as follows

Time (yrs)	*Probability of default*	*Recovery Rate*	*Expected Payoff*	*Discount Factor*	*PV of Expected Payment*
0.5	0.0300	0.3	0.0210	0.9656	0.0203
1.5	0.0291	0.3	0.0204	0.9003	0.0183
2.5	0.0282	0.3	0.0198	0.8395	0.0166
3.5	0.0274	0.3	0.0192	0.7827	0.0150
4.5	0.0266	0.3	0.0186	0.7298	0.0136
Total					0.0838

Expected payoff = PD x (1-RR) x notional
Discount factor calculations are on continuous basis.
PV of Expected Payment = Expected Payoff x Discount Factor.

The present value of accrual payments made in the event of default, are

Time (yrs)	*Probability of default*	*Expected Accrual Payment*	*Discount Factor*	*PV of Expected Accrual Payment*
0.5	0.0300	0.0150s	0.9656	0.0145s
1.5	0.0291	0.0146s	0.9003	0.0131s
2.5	0.0282	0.0141s	0.8395	0.0118s
3.5	0.0274	0.0137s	0.7827	0.0107s

4.5	0.0266	0.0133s	0.7298	0.0097s
Total				0.0598s

The credit default swap spread s is given by:

$$3.7364s + 0.0598s = 0.0838$$

payment rate is s per year + present value of accrual payments = present value of the expected payoffs.

It is 0.0221 or 221 basis points.

2 - If the credit default swap spread is 150 basis points, the value of the swap to the buyer of protection is:

$$0.0838 - (3.7364 + 0.0598) \times 0.0150 = 0.0269$$

per dollar of notional principal.